Engineering Notes

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AIAA 81-4289

Performance Prediction for Light Airplanes

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I. Introduction

Oswald's method[1] for estimating the performance of airplanes with fixed-pitch propellers is extended to modern airplanes with variable-pitch propellers. Oswald approximated the power available by the empirical ratio $(V/V_m)^{0.55}$ when the fixed-pitch propeller was selected to produce its "peak" (P_m) power output at the maximum level flight velocity at sea level. However, a parabolic variation of the thrust with the flight velocity makes it possible to develop explicit relations for the maximum level flight speed and the maximum rate of climb at any altitude up to the absolute ceiling, which is also predicted by an explicit relation. Finally, it is shown that the constant power available method generally predicts a much too high rate of climb.

II. Drag Polar and Parabolic Thrust Variation

THE well-known drag polar diagram was first applied in a systematic way to aircraft performance calculations by Oswald[1] who expressed it in the following manner

$$C_D = C_{D_e} + C_L^2/\pi \mathit{Æ} e \tag{1}$$

by selecting C_{D_e} and e so as to form a parabola that would give the best fit to actual wind tunnel or flight test drag polar data. The purpose of the present paper is to review current methods for determining the constants C_{D_e} and e, show what errors can occur if the limitations of the best-fit parabola are exceeded, and finally show how the performance calculations are greatly simplified by representing the propulsive thrust by a parabola given by

$$T(V,\sigma) = a(\sigma) - b(\sigma)V^2 = W[\phi(\sigma)\tau - \sigma V^2/\nu] \tag{2}$$

von Mises[2] proved that Eq. (2) is the correct theoretical variation of the available propulsive thrust from any fixed-pitch propeller where the decrease in static thrust (a) with altitude is approximated by the empirical relation introduced by Bairstow (Ref. 3), $\phi(\sigma) = (\sigma - C)/(1-C)$, with $C=0.12$, however, for a two-bladed metal propeller with a fixed-pitch, von Mises[2] found the best approximation with $C=0.13$.

III. Oswald's Equations for Aircraft Drag Force

In applying Eq. (1), both Oswald[1] and von Mises[2] made the usual assumption prevalent at that time that the glide or climb angle (θ) was sufficiently small so that $\cos\theta \approx 1$ and $L \approx W$. However, large climb angles and a finite angle of attack (α) between the thrust and velocity vector require

$$W\cos\theta - T\sin\alpha = L = (\sigma/2)\rho_0 S V^2 C_L$$
$$T\cos\alpha - W\sin\theta = D = (\sigma/2)\rho_0 S V^2 C_D \tag{3}$$

When the parabolic drag variation predicted by Eq. (1) is introduced into Eq. (3), we obtain

$$D = AV^2 + BV^{-2} = W\left(\frac{\sigma V^2}{\lambda_p} + \frac{\lambda_s}{\sigma V^2}\right) \tag{4}$$

where

$$A = \frac{\sigma}{2}\rho_0 S C_{D_e} \qquad B = \frac{(W\cos\theta - T\sin\alpha)^2}{(\sigma/2)\rho_0 S \pi \mathit{Æ} e} \tag{5}$$

The terms λ_p and λ_s are independent of σ and were introduced by Oswald for $\alpha \approx 0$:

$$\lambda_p = \frac{\sigma W}{A} = \frac{2W}{\rho_0 S C_{D_e}} \qquad \lambda_s = \frac{\sigma B}{W} = \frac{2W\cos^2\theta}{\rho_0 S \pi \mathit{Æ} e} \tag{6}$$

Figure 1 presents graphs of Eq. (1) plotted vs C_L^2 so the parabolic curve fitting procedure is replaced by a straight line approximation. The data for the single jet engine airplane is taken from Dommasch[4] and it is seen that $C_{D_e} = 0.014$ and $\pi \mathit{Æ} e = 20$ give a satisfactory approximation for $0.14 < C_L < 0.84$, which includes the range for level flight at maximum L/D $(C_L \approx 0.5)$, and the maximum rate of climb $(C_L \approx 0.14)$. However, the C_D value for level flight at minimum power is underestimated by nearly 10% because $C_{L_{mp}} = 0.917$ and the given approximation is not applicable at these higher C_L.

The upper curve in Fig. 1 that is approximated by the straight line defined by $C_{D_e} = 0.03$ and $\pi \mathit{Æ} e = 17$ represents a single propeller airplane with a fixed landing gear similar to a Cessna 120 or a Piper PA-22 ($\mathit{Æ} \approx 6$; $e \approx 0.9$). It is seen that the approximation predicts C_D values within 5% for $0.4 < C_L < 1$, but is totally inadequate for level flight at minimum power when $C_{L_{mp}} = 1.24$.

IV. Performance Calculation by Explicit Relations

The power available from a fixed-pitch propeller resembles the cubic curve shown in Fig. 2, which can be approximated by the parabolic thrust variation from Eq. (2) as

$$P = TV = aV - bV^3 = W(\phi\tau V - \sigma V^3/\nu) \tag{7}$$

The maximum peak of this cubic occurs at any desired V_m at sea level by evaluating $\mathrm{d}P/\mathrm{d}V = 0$ so as to obtain

$$V_m = (a/3b)^{1/2} \qquad b = a/3V_m^2 = P_m/2V_m^3 \tag{8}$$

$$P_m = T_m V_m = (2a/3)V_m \qquad a = 3P_m/2V_m \tag{9}$$

At any altitude Eq. (2) then may be written as

$$\frac{T}{W} = \phi\tau - \sigma V^2/\nu = \frac{P_m}{WV_m}\left[\frac{3}{2}\phi - \frac{\sigma}{2}\left(\frac{V}{V_m}\right)^2\right] \tag{10}$$

Received July 28, 1980; revision received May 19, 1981.

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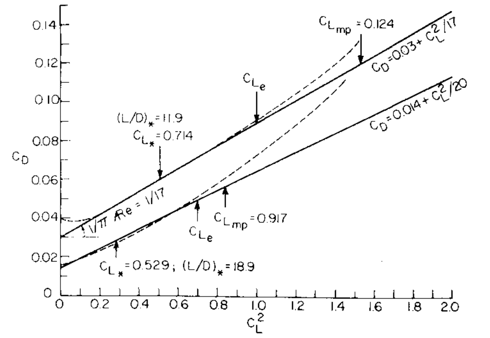

Fig. 1 Application of Eq. (1) to the experimental data for a propeller airplane ($C_{D_e}=0.03$), and a jet airplane ($C_{D_e}=0.014$).

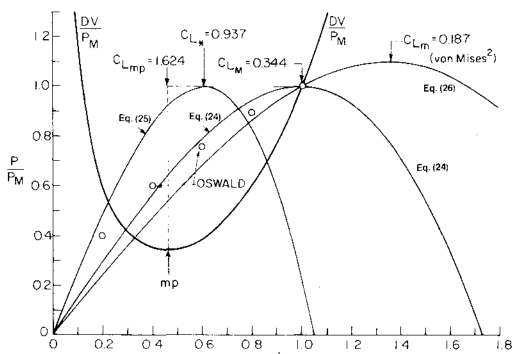

Fig. 2 Variation of power available (TV) and power required (DV) with velocity in terms of the power required (P_M) at maximum velocity (V_M) for the airplane data given by von Mises.[2]

Equating Eqs. (2) and (4) shows that the maximum level flight speed is given by $T=D$ when $V=V_M$, so that

$$a-(A+b)V_M^2-BV_M^{-2}=0$$

$$V_M^2=\frac{a}{2(A+b)}\left[1+\left(1-\frac{4B(A+b)}{a^2}\right)^{1/2}\right]$$

$$=\frac{\phi\tau}{2\sigma(1/\lambda_p+1/\nu)}\left[1+\left(1-\frac{4\lambda_s(1/\lambda_p+1/\nu)}{\phi^2\tau^2}\right)^{1/2}\right] \quad (11)$$

For a jet airplane one has $\phi\approx\sigma(C\approx 0)$ and $1/\nu\approx 0$ so that Eq. (11) reduces to the equations given by Dommasch[4] and Houghton and Brock[5] for the maximum level flight speed of a jet airplane with $T=\sigma T_0=a=\sigma\tau W$, when the thrust is assumed independent of the flight speed.

It is important to note that the minus (−) algebraic value which can be assigned to the square root term in Eq. (11) defines a minimum speed that is physically impossible at low altitudes since it corresponds to $C_L>C_{L_{max}}$. However, as one approaches the absolute ceiling it can approximate the correct minimum velocity as long as $C_L<C_{L_e}$, where C_{L_e} is the upper limit of the approximation provided by Eq. (1) as shown in Fig. 1. At the absolute ceiling we have $V_M=V_{min}=V_h$ so the square root term in Eq. (11) must vanish, and therefore the maximum attainable altitude (h) is defined by

$$a_h=2[B(A+b)]^{1/2} \qquad \phi_h=(2/\tau)[\lambda_s(1/\lambda_p+1/\nu)]^{1/2}$$

$$V_h^2=\frac{a_h}{2(A+b)}=\frac{\phi_h\tau}{2\sigma_h(1/\lambda_p+1/\nu)}=\frac{1}{\sigma_h}\left(\frac{\lambda_s}{1/\lambda_p+1/\nu}\right)^{1/2} \quad (12)$$

The flight velocity (V_w) that produces the maximum rate of climb, or maximum vertical velocity component, $w=V_w \sin\theta_w$, can be obtained by the following procedure based on Eqs. (2-4).

$$\frac{d}{dV}(T\cos\alpha-D)V=0=(a\cos\alpha)-3(A+b\cos\alpha)V_w^2+BV_w^{-2}$$

$$V_w^2=\frac{a\cos\alpha}{6(A+b\cos\alpha)}\left[1+\left(1+\frac{12B(A+b\cos\alpha)}{(a\cos\alpha)^2}\right)^{1/2}\right] \quad (13)$$

If $\cos\alpha\approx 1$, then the Oswald coefficients can be introduced

$$V_w^2=\frac{\phi\tau}{6\sigma(1/\lambda_p+1/\nu)}\left[1+\left(1+\frac{12\lambda_s(1/\lambda_p+1/\nu)}{\phi^2\tau^2}\right)^{1/2}\right] \quad (14)$$

and maximum climb rate (w) can now be calculated as follows

$$\left(\frac{dh}{dt}\right)_{max}=w=\frac{T-D}{W}V_w=V_w\sin\theta_w$$

$$\sin\theta_w=\frac{T-D}{W}=\frac{1}{W}[a-(A+b)V_w^2-BV_w^{-2}]$$

$$=\frac{2}{3}\frac{a}{W}\left[2-\left(1+\frac{12B(A+b)}{a^2}\right)^{1/2}\right]=\frac{w}{V_w}$$

$$=\frac{2}{3}(\phi\tau)\left[2-\left(1+\frac{12\lambda_s(1/\lambda_p+1/\nu)}{\phi^2\tau^2}\right)^{1/2}\right] \quad (15)$$

For a jet airplane with $a=\sigma T_0$, $b=0$, and $\cos\alpha=1$, Eq. (13) reduces to the relations given by Dommasch[4] and Houghton and Brock.[5] A considerable simplification and a better physical interpretation of these results for a jet airplane can be obtained if one introduces the well-known relations for the maximum lift-to-drag ratio based on Eqs. (1) and (4), e.g., see Ref. 1, 2, or 5, namely,

$$C_{D_*}=2C_{D_e}, \quad C_{L_*}=(\pi A\!\!R eC_{D_e})^{1/2}, \quad \left(\frac{L}{D}\right)_*=\frac{1}{2}\left(\frac{\pi A\!\!R e}{C_{D_e}}\right)^{1/2}$$

$$D_*=2(AB)^{1/2}=2W\left(\frac{\lambda_s}{\lambda_p}\right)^{1/2}=\frac{W\cos\theta}{(L/D)_*}$$

$$V_*^2=\left(\frac{B}{A}\right)^{1/2}=\frac{(\lambda_p\lambda_s)^{1/2}}{\sigma}=\frac{W\cos\theta}{(1/2)\sigma\rho_0 SC_{L*}}=\frac{D_*}{\sigma\rho_0 SC_{D_e}} \quad (16)$$

Then Eq. (13) can be written for $\cos\alpha\approx 1$ as

$$\left(\frac{V_w}{V_*}\right)^2=\frac{\sigma T_0}{3D_*}\left\{1+\left[1+3\left(\frac{D_*}{\sigma T_0}\right)^2\right]^{1/2}\right\} \quad (17)$$

Similarly, the maximum velocity of a jet aircraft in steady level flight can be estimated from Eq. (11) as

$$\left(\frac{V_M}{V_*}\right)^2=\frac{\sigma T_0}{D_*}\left\{1+\left[1-\left(\frac{D_*}{\sigma T_0}\right)^2\right]^{1/2}\right\} \quad (18)$$

This gives the interesting relation that the absolute ceiling is attained when $T=\sigma_h T_0=D_*$, where D_* is now the constant level flight minimum drag that is the same at all altitudes, and is defined by Eq. (16) with $\cos\theta=1$. Consequently, if Eq. (1) is applicable then at the absolute ceiling

$$V_h^2=\frac{T_0}{\rho_0 SC_{D_e}}=\frac{D_*}{\sigma_h\rho_0 SC_{D_e}}=V_*^2=V_M^2=V_{w=0}^2 \quad (19)$$

Therefore any jet aircraft whose thrust is independent of the flight velocity so that Eq. (2) can be used with $a=\sigma T_0$ and $b=0$ reaches its absolute ceiling when $\sigma_h = D_*/T_0$ at the steady level flight velocity V_* and the corresponding C_{L*}, which is generally less than C_{L_e}, as indicated in Fig. 1. The numerical results from Eqs. (17-19) are in exact agreement with the jet aircraft performance calculations given by Houghton and Brock,[5] and by Anderson.[6] However, Eq. (17) predicts that the flight velocity (V_w) for the maximum rate of climb continually increases with altitude until $V_w = V_*$ at absolute ceiling, whereas it is known that all jet aircraft capable of flight Mach numbers greater than 0.75 must decrease V_w at the higher altitudes since V_* approaches the speed of sound. In this case, compressibility effects have limited the application of Eq. (1), which provides a satisfactory approximation only when the drag rise due to these compressibility effects can be neglected.

The flight velocity (V_θ) that produces the maximum climb angle is given by

$$\frac{\mathrm{d}}{\mathrm{d}V}(T\cos\alpha - D) = 0 = -2(A + b\cos\alpha)V_\theta + 2BV_\theta^{-3}$$

$$V_\theta^2 = \left(\frac{B}{A + b\cos\alpha}\right)^{1/2} \leq V_*^2 = \left(\frac{B}{A}\right)^{1/2} \tag{20}$$

Therefore, the maximum climb angle is given by

$$(\sin\theta)_{\max} = \frac{a\cos\alpha}{W} - \frac{2}{W}\left(B(A + b\cos\alpha)\right)^{1/2}$$

$$= \frac{a\cos\alpha}{W} - \left[\left(\frac{D_*}{W}\right)^2 + \frac{4Bb\cos\alpha}{W^2}\right]^{1/2} \tag{21}$$

For the previously considered jet airplane, these relations reduce to flight at the maximum lift-to-drag ratio as given by Eq. (16), with a maximum climb angle obtained from the obvious relation

$$(\sin\theta)_{\max} = \sigma(T_0/W)\cos\alpha - (D_*/W) \tag{22}$$

For propeller aircraft $b>0$ and $V_\theta < V_*$ so one must check that $C_{L_\theta} < C_{L_e}$ to make certain that Eqs. (20) and (21) are applicable. It is interesting to note that at its absolute ceiling any propeller aircraft having $\cos\alpha \approx 1$, has

$$V_{\min} = V_M = V_w = V_\theta = V_h = \left(\frac{B}{A+b}\right)^{1/4} < V_* = \left(\frac{B}{A}\right)^{1/4} \tag{23}$$

Consequently, Eqs. (1) and (2) predict that a fixed-pitch propeller aircraft does not have sufficient power available to fly at its maximum lift-to-drag ratio when it is at or near its absolute ceiling.

It will now be shown that a very useful relation for predicting the power available for modern propeller-driven light aircraft is given by rewriting Eq. (10) as

$$\frac{TV}{P_m} = \frac{3}{2}\phi\left(\frac{V}{V_m}\right) - \frac{\sigma}{2}\left(\frac{V}{V_m}\right)^3 = \frac{TV}{T_m V_m} \tag{24}$$

For a fixed-pitch propeller, Oswald[1] has indicated that for most aircraft the best performance is obtained when the maximum or peak-power (P_m) is available at the maximum velocity (V_M) for steady level flight at sea level as shown by Eq. (24) in Fig. 2 for the aircraft used by von Mises.[2] Figure 2 indicates that the best engine-propeller combination would have the peak-power available near, but not above, the highest possible level flight velocity. For example, if the fixed-pitch propeller was selected so that the peak-power (P_m) occurred at the velocity (V_*) corresponding to the aircraft's

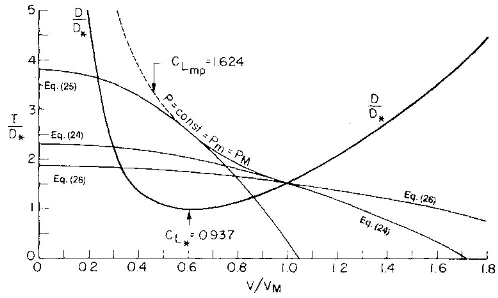

Fig. 3 Variation of the thrust (*T*) and the drag (*D*) with the velocity from Fig. 2.

maximum lift-to-drag ratio as shown in Fig. 2, then Eq. (24) would be replaced by

$$\frac{TV}{P_m} = \frac{3}{2}\phi\left(\frac{V}{V_*}\right) - \frac{\sigma}{2}\left(\frac{V}{V_*}\right)^3 = \frac{TV}{T_* V_*} \tag{25}$$

Now we have a finite rate of climb at peak power because $T_* > D_*$, whereas we previously had a zero rate of climb at peak power since $T_m = D_m$ as shown in Fig. 3 when $V = V_m = V_M$. On the other hand, there is no advantage in having the peak power occur after the maximum possible level flight velocity, as shown in Fig. 2 by Eq. (26) for the fixed-pitch propeller selected by von Mises.[2] The first disadvantage is that a larger engine is required in order to produce the greater peak power. The second disadvantage is that there is less power available to climb at all speeds below the maximum velocity for steady level flight at sea level. In this case one must use the following relation to express the power available in terms of the actual power (P_M) required at the maximum possible velocity (V_M) for steady level flight at sea level:

$$\frac{TV}{P_M} = \left(\frac{L}{D}\right)_M \left[\phi\tau\left(\frac{V}{V_M}\right) - \sigma\left(\frac{V_M^2}{\nu}\right)\left(\frac{V}{V_M}\right)^3\right] = \frac{TV}{T_M V_M} \tag{26}$$

where

$$\left(\frac{L}{D}\right)_M = \frac{2(L/D)_*}{(V_M/V_*)^2 + (V_*/V_M)^2} \tag{27}$$

is the lift-to-drag ratio at the maximum level flight velocity (V_M) at sea level.

It should be noted that Figs. 2 and 3 indicate that a method for analyzing the performance of aircraft with a variable-pitch propeller would be to use Eq. (24) for all diving maneuvers when $V \geq V_m = V_M$ and then assume a constant available power P_m in the velocity range $V_0 \leq V \leq V_m$, where $V_0 \geq V_*$ is selected so that Eqs. (12-15) would predict a sufficiently large V_w, so that $C_{L_w} < C_{L_e}$. Figure 2 clearly shows that by connecting the peak powers corresponding to Eqs. (24) and (25) with the solid straight line representing a thrust power independent of speed, one has a satisfactory approximation to the typical variation of the power available from a constant rpm variable-pitch propeller.

Frequently the approximation is made that the available power remains constant independent of the flight speed in the entire flight range, rather than in the limited region indicated by the solid horizontal line in Fig. 2. The dotted line shows that the maximum excess power, and therefore the maximum rate of climb, occurs when the required power (DV) is a minimum (e.g., see Ref. 2 or 5) so that $C_{L_W} = C_{L_{mp}} = \sqrt{3}\, C_{L*}$. However, in the usual case $C_{L_{mp}} > C_{L_e}$ as indicated in Fig. 1 so that the actual $C_D > 4C_{D_e}$ and the climb rate is overestimated. This was first pointed out by Oswald[1] and is clearly evident in Fig. 2 since $C_{L_{mp}} = 1.624 > C_{L_{\max}}$. It should also be noted by the dotted line in Fig. 3 how the available thrust can be greatly over-estimated at the lower speeds.

However, if the power available does remain constant for speeds as low as V_{mp}, and one has $C_{L_{mp}} < C_{L_e}$, then the maximum climb rate is given by

$$V_w = V_{mp} = (\lambda_p \lambda_s / 3)^{1/4} / \sqrt{\sigma}, \qquad w = [\phi P_m - (mp)_0 / \sqrt{\sigma}] / W$$

$$(mp)_0 = (4W/3^{3/4})(\lambda_s^{3/4} \lambda_p^{-1/4}) = (2/3^{1/4}) D_*(V_*)_{\sigma=1} \qquad (28)$$

where $(mp)_0$ is the minimum power required at sea level. The absolute ceiling is given by the numerical solution of $\phi(\sigma)$ and $\sigma(h)$ from the following equation for $w=0$

$$\phi_h P_m = (mp)_0 / \sqrt{\sigma_h} \qquad (29)$$

For convenience, Anderson[6] assumed $\phi = \sigma$ in his numerical calculations, which of course are in exact agreement with Eq. (28) and now Eq. (29) reduces to a simple explicit relation for the value of σ at the absolute ceiling, namely,

$$\sigma_h = [(mp)_0 / P_m]^{2/3} \qquad (30)$$

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AIAA 81-4290

Influence of Landing Gear Flexibility on Aircraft Performance During Ground Roll

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Nomenclature

a = acceleration during ground roll
C_D = aircraft drag coefficient, including ground effect
D = total drag of the aircraft, including ground effect
g = gravitational acceleration
I = inertia force on the aircraft during ground roll
k_0, k_1, k_2, k_3 = landing gear deflection constants defined in Eqs. (4) and (5)
L_{HT} = horizontal tail lift
L_{WB} = aircraft lift without tail contribution
MAC = mean aerodynamic chord
M_D = moment about the main wheel contact point (mwcp) due to drag
M_{HT} = moment about mwcp due to tail lift
M_I = moment about mwcp due to inertia
M_L = moment about mwcp due to aircraft lift
M_{R_N} = moment about mwcp due to ground reaction at the nose gear
M_T = moment about mwcp due to thrust
M_W = moment about mwcp due to aircraft weight
M_0 = aerodynamic pitching moment about the aircraft aerodynamic center
R_M = ground reaction on the main landing gear
R_N = ground reaction on the nose landing gear
T = thrust
W = aircraft takeoff weight
Y_t, Y_s = tire deflection and wheel axle travel defined in Eqs. (4) and (5)
μ = coefficient of ground (runway) friction (~0.035)
Θ = ground incidence

Received Oct. 27, 1980; revision received March 18, 1981.

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Introduction

IT is generally assumed (see, for example, Ref. 1) that the attitude of the aircraft remains constant during ground roll and, consequently, C_L, C_D and other aerodynamic coefficients also remain constant during this phase. However, the flexibility of the landing gear system appreciably alters the attitude of the aircraft during ground roll. An analysis of the influence of landing gear deflection characteristics on the aircraft performance on the ground up to rotation is presented in this paper.

Problem Formulation

In order to rotate the aircraft about its main wheels, the horizontal tail has to overcome moments produced by the following forces about the main wheels: aircraft weight; thrust; inertia; ground reactions on the wheels; and the aerodynamic forces and moments, that is, lift, drag, and pitching moment of the aircraft. These are shown schematically in Fig. 1. A quasisteady dynamic equilibrium state has been assumed. The following simplifying assumptions have also been made: 1) calm air conditions; 2) C_{M_0} and C_D of the tail unit are negligible; 3) the aircraft lift and drag acting through the aerodynamic center are respectively normal and parallel to the horizontal ground plane. The ground incidence Θ is defined as the angle made by the mean aerodynamic chord of the wing with respect to the ground plane. The following equations for force and moment balance determine the quasiequilibrium conditions for the aircraft during its ground roll:

Forces normal to the ground:

$$L_{WB} + L_{HT} + (R_N + 2R_M) + T\sin\Theta = W \qquad (1)$$

Forces parallel to the ground:

$$T\cos\Theta - D - \mu(R_N + 2R_M) = (W/g)a = I \qquad (2)$$

Moment about the main wheel contact point with the ground:

$$M_W + M_0 + M_T + M_L + M_D + M_I + M_{R_N} = M_{HT} \qquad (3)$$

In Eqs. (1-3) the unknowns are R_N, R_M, and L_{HT}. All others are known for a given aircraft attitude and speed. However, the aircraft attitude itself depends on the deflections of the landing gears which, in turn, are functions of R_N and R_M. It is possible to express the landing gear deflection in the vertical direction as a function of normal reaction as follows:

Tires:

$$Y_t = k_t R \qquad (4)$$

Wheel axle travel:

$$Y_s = k_0 + k_1 R + k_2 R^2 \qquad (5)$$